

Geometric phase and neutrino mass hierarchy problem

Khushboo Dixit,^{1,*} Ashutosh Kumar Alok,^{1,†} Subhashish Banerjee,^{1,‡} and Dinesh Kumar^{2,§}

¹*Indian Institute of Technology Jodhpur, Jodhpur 342011, India*

²*Department of Physics, University of Rajasthan, Jaipur 302004, India*

We study the geometric phase for neutrinos at various man-made facilities, such as the reactor and accelerator neutrino experiments. The analysis is done for the three flavor neutrino scenario, in the presence of matter and for general, noncyclic paths. The geometric phase is seen to be sensitive to the CP violating phase in the leptonic sector and the sign ambiguity in Δ_{31} . We find that for experimental facilities where the geometric phase can complete one cycle, all geometric phase curves corresponding to different values of CP violating phase, converge to a single point, called the *cluster point*. There are two distinct cluster points for positive and negative signs of Δ_{31} . Thus measurement of geometric phase in these experimental set-ups would help in resolving the neutrino mass hierarchy problem.

I. Introduction

The phenomenon of neutrino oscillations, which is experimentally well established [1–7], has emerged as one of the most prominent areas of research over the past decade. Not only it has provided evidence of physics beyond the Standard Model of particle physics but has also provided deep insight into the understanding of mass. The present and future planned facilities in neutrino sector aims at resolving yet many unsolved puzzles, such as observation of CP violation in the leptonic sector, resolving mass hierarchy problem, probing sterile neutrinos, resolving Dirac vs Majorana neutrinos, probing ultra high energy neutrinos, dark matter searches. Recently neutrinos have been suggested as a tool to probe foundational issues in quantum mechanics [8–11].

The CP violation in the quark sector has been measured and well established in the standard model of particle physics [12]. The CP violation the quark mixing matrix can arise due to the complex Yukawa couplings. A relative CP violating phase in the vacuum expectation values of the Higgs fields can also give rise to the CP violation in the quark sector. An analogous mechanism is expected to induce CP violation in the leptonic sector which has yet not been observed. The neutrino oscillation experiments provides a platform to measure the CP violation in the leptonic mixing matrix known as the PMNS matrix [13]. The presently running and planned experiments in neutrino sector also aims to resolve the mass hierarchy problem, i.e., whether the mass appearing in ν_3 -eigenstate of neutrino is the lightest or the heaviest of the three masses. In this work we study

these issues in the context of geometric phase (GP) in neutrino oscillations.

The concept of GP exists in the standard Pontecorvo formulation of neutrino oscillations [14]. The adiabaticity condition needs to be satisfied in order to have Berry phase. This condition is provided by varying the matter density. The Berry phase in the context of two flavor neutrino oscillations was calculated in Ref. [14]. The possibility of observation of Berry phase in long baseline neutrino oscillation experiments was discussed in [15]. In order to extract Berry phase, one needs to vary the oscillation distance L while maintaining the cyclicity condition [15]. Thus one has to measure this phase after the state has undergone a complete cycle with time period T . In neutrino oscillations, this period is relatively long so, one needs to place the detector at sufficiently large distance from the source to observe the Berry phase. The generalization of GP to noncyclic paths was made shortly after Berry's discovery involving closed cyclic paths, see for example [16]. The noncyclic phase could be easier to observe because the cyclicity constraint is removed [34]. Recently, noncyclic GP has been studied to reveal whether the neutrino is Dirac or Majorana particle [17, 18]. In [19] the topological phase in two-flavor neutrino oscillations was discussed by using the analogy between the two-flavor state and the polarization states in optics.

GP has a rich and interesting history. It was introduced by Berry [20], in the quantum scenario, for the case of cyclic adiabatic evolution. The connection of this work to the earlier work of Pancharatnam [21] related to the interference of polarized light was made in [22]. Berry's phase was shown to be a consequence of the holonomy in a line bundle over parameter space [23], bringing out the geometric nature of the phase. Generalization of Berry's work to non-adiabatic evolution was made in [24] and to the case of non-cyclic evolution in [25]. The quantum kinematical aspect of GP was established in [26]. A kinematic approach to GP for mixed states un-

*Electronic address: dixit.1@iitj.ac.in

†Electronic address: akalok@iitj.ac.in

‡Electronic address: subhashish@iitj.ac.in

§Electronic address: dinesh@uniraj.ac.in

dergoing nonunitary evolution was proposed in [27]. In the context of open quantum systems, GP has been studied for a variety of systems, see for example, [28–32]. In [33] Abelian and non-Abelian geometric phases for open quantum systems was considered.

In this work we study non-cyclic GP in the context of three flavor neutrino oscillations in the presence of matter and CP violating effects. We discuss how GP can be used to distinguish between the normal and inverted hierarchy of neutrino masses. We analyze GP in the context of two reactor neutrino experiments, Daya-Bay & RENO and two accelerator experiments, T2K & NO ν A. We find that by measuring the GP in T2K, Daya-Bay and RENO experimental set-ups one can distinguish effects of normal and inverted mass hierarchy for all values of CP violating phase.

The paper is arranged as follows. After the introduction, we provide theoretical expressions for geometric phase, both in vacuum and in presence of matter. In Sec. III, we present our results for different experimental set-ups. Finally, in Sec. IV, we present our conclusions.

II. Noncyclic geometric phase

Let us consider a state $|\chi(0)\rangle$ which evolves to a state $|\chi(t)\rangle$, after a certain time t . Then the scalar product

$$\langle\chi(0)|\exp\left[\frac{i}{\hbar}\int_0^t\langle E\rangle(t')dt'\right]|\chi(t)\rangle, \quad (1)$$

can be written as $r e^{i\beta}$, where r is real number and angle β is the noncyclic phase due to the evolution from $|\chi(0)\rangle$ to $|\chi(t)\rangle$ [34]. We now consider three flavor oscillating neutrino states, $|\nu_\alpha(0)\rangle = U_{\alpha a}|\nu_a\rangle$, where $|\nu_\alpha(0)\rangle$ is initial flavor state ($\alpha = e, \mu, \tau$) and $|\nu_a\rangle$ are mass eigenstates ($a = 1, 2, 3$). $U_{\alpha a}$ are the elements of the PMNS mixing matrix U given by

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & e^{i\delta}s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{-i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} \quad (2)$$

Here $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$ and δ is the CP violating phase.

The time evolution of $|\nu_\alpha\rangle$ in the flavor basis is given by

$$|\nu_\alpha(t)\rangle = U U_m(t) U^{-1} |\nu_\alpha(0)\rangle = U_f(t) |\nu_\alpha(0)\rangle. \quad (3)$$

$$U_m = \begin{pmatrix} e^{-i\omega_1 t} & 0 & 0 \\ 0 & e^{-i\omega_2 t} & 0 \\ 0 & 0 & e^{-i\omega_3 t} \end{pmatrix} \quad (4)$$

and $U_f(t) = U U_m(t) U^{-1}$.

To calculate the noncyclic phase generated during the time evolution of, say $|\nu_\mu(0)\rangle$ to $|\nu_\mu(t)\rangle$, we define a new state $|\tilde{\nu}_\mu(t)\rangle$, given by

$$|\tilde{\nu}_\mu(t)\rangle = \exp\left[i\int_0^t\langle E\rangle(t')dt'\right]|\nu_\mu(t)\rangle, \quad (5)$$

where $\langle E\rangle(t') = \langle\nu_\mu(t')|i\partial_t|\nu_\mu(t')\rangle$, and

$$|\nu_\mu(t')\rangle = U_{f12}(t')|\nu_e\rangle + U_{f22}(t')|\nu_\mu\rangle + U_{f32}(t')|\nu_\tau\rangle, \quad (6)$$

with $\langle\nu_\mu(0)|\nu_\mu(t')\rangle = U_{f22}(t')$, $U_{fab}(t')$ are $a \times b$ elements of the evolution operator U_f , ($a, b = 1, 2, 3$). Then the scalar product is given by

$$\langle\nu_\mu(0)|\tilde{\nu}_\mu(t)\rangle = \exp\left[i\int_0^t\langle E\rangle(t')dt'\right]U_{f22}(t') \equiv r_{\mu\mu}e^{i\beta_{\mu\mu}}, \quad (7)$$

so that the noncyclic phase due to evolution from $|\nu_\mu(0)\rangle$ to $|\nu_\mu(t)\rangle$ will be the phase angle $\beta_{\mu\mu}$. Similarly $\beta_{\mu e}$, will be evaluated by the component $\langle\nu_\mu(0)|\tilde{\nu}_e(t)\rangle$ as

$$|\tilde{\nu}_e(t)\rangle = \exp\left[i\int_0^t\langle E\rangle(t')dt'\right]|\nu_e(t)\rangle, \quad (8)$$

and

$$\langle\nu_\mu(0)|\tilde{\nu}_e(t)\rangle = \exp\left[i\int_0^t\langle E\rangle(t')dt'\right]U_{f21}(t') \equiv r_{\mu e}e^{i\beta_{\mu e}}, \quad (9)$$

where $\langle E\rangle(t') = \langle\nu_e(t')|i\partial_t|\nu_e(t')\rangle$, both in Eqs. (8) and (9).

A. Non cyclic geometric phase in vacuum

Using the definition of non-cyclic GP from the previous section, the theoretical expressions of GP, such as β_{ee} , $\beta_{e\mu}$, $\beta_{\mu\mu}$ and $\beta_{\mu e}$, are [34]

$$\beta_{ee} = [2s_{12}^2 c_{13}^2 + 2qs_{13}^2 - 1](\phi \frac{L}{C}) + \tan^{-1} \frac{\cos(2\theta_{12})c_{13}^2 \sin \phi \frac{L}{c} - s_{13}^2 \sin[(2q-1)\phi \frac{L}{c}]}{c_{13}^2 \cos \phi \frac{L}{c} + s_{13}^2 \cos[(2q-1)\phi \frac{L}{c}]}, \quad (10)$$

$$\beta_{e\mu} = [\cos(2\theta_{12})(c_{23}^2 - s_{13}^2 s_{23}^2) + c_{13}^2 s_{23}^2 (2q-1) - s_{13} \sin(2\theta_{12}) \sin(2\theta_{23}) \cos(\delta)](\phi \frac{L}{C}) + \tan^{-1} \frac{s_{13} s_{23} [s_{12}^2 \sin(\phi \frac{L}{c} + \delta) - c_{12}^2 \sin(\phi \frac{L}{c} - \delta) - \sin \zeta_+] - c_{23} \sin(\phi \frac{L}{c}) \sin(2\theta_{12})}{\sin(\theta_{13}) \sin(\theta_{23}) [\cos \zeta_+ - s_{12}^2 \cos(\phi \frac{L}{c} + \delta) - c_{12}^2 \cos(\phi \frac{L}{c} - \delta)]}, \quad (11)$$

$$\beta_{\mu\mu} = [\cos(2\theta_{12})(c_{23}^2 - s_{13}^2 s_{23}^2) + c_{13}^2 s_{23}^2 (2q-1) - s_{13} \sin(2\theta_{12}) \sin(2\theta_{23}) \cos \delta](\phi \frac{L}{c}) + \tan^{-1} \frac{s_{13} \sin(2\theta_{12}) \sin(2\theta_{23}) \cos \delta \sin(\phi \frac{L}{c}) - (c_{23}^2 - s_{23}^2 s_{13}^2) \cos(2\theta_{12}) \sin(\phi \frac{L}{c}) - c_{13}^2 s_{23}^2 \sin \Gamma}{(c_{23}^2 + s_{13}^2 s_{23}^2) \cos(\phi \frac{L}{c}) + c_{13}^2 s_{23}^2 \cos \Gamma}, \quad (12)$$

$$\beta_{\mu e} = (2s_{12}^2 c_{13}^2 + 2qs_{13}^2 - 1)(\phi \frac{L}{c}) + \tan^{-1} \frac{s_{13} s_{23} [s_{12}^2 \sin(\phi \frac{L}{c} - \delta) - c_{12}^2 \sin(\phi \frac{L}{c} + \delta) - \sin \psi_-] - c_{23} \sin(\phi \frac{L}{c}) \sin(2\theta_{12})}{\sin(\theta_{13}) \sin(\theta_{23}) [\cos \psi_- - s_{12}^2 \cos(\phi \frac{L}{c} - \delta) - c_{12}^2 \cos(\phi \frac{L}{c} + \delta)]}, \quad (13)$$

where $q = (\omega_3 - \omega_1)/(\omega_2 - \omega_1) = (\Delta_{32}/\Delta_{21}) + 1$; $\phi = \Delta_{21}/4E\hbar$; $\Gamma = (2q-1)(\phi L/c)$ and $\psi_{\pm} = (2q-1)(\phi L/c) \pm \delta$. Further, L is the propagation length and E is the neutrino energy. For the cyclic case scenario, q would be a rational number with the cyclic period $t = 2\pi/(\omega_1 - \omega_2)$ [14, 34].

In our analysis, the values of mixing angles, θ_{ij} , and mass square differences, $\Delta_{ij} = m_i^2 - m_j^2$ (m_i and m_j are the masses of the neutrino mass eigenstates ν_i and ν_j , respectively), used are $\theta_{12} = 33.48^\circ$, $\theta_{13} = 8.5^\circ$, $\theta_{23} = 42.3^\circ$, $\Delta_{21} = 7.5 \times 10^{-5} eV^2$, $\Delta_{31} (\approx \Delta_{32}) = \pm 2.457 \times 10^{-3} eV^2$. The plots of β_{ee} and $\beta_{e\mu}$ with respect to L (for two cycles) for neutrino energy $E = 1$ MeV for different values of δ are shown in Fig. 1. As can be seen from the figure, for $\beta_{\mu\mu}$, there is mild dependence on δ whereas $\beta_{\mu e}$ is sensitive to δ . Similarly, we find that $\beta_{e\mu}$ is sensitive to the CP phase whereas β_{ee} is not. Therefore in order to study the effect of CP violating phase on GP, we consider $\beta_{e\mu}$ and $\beta_{\mu e}$ in this work.

B. Non cyclic geometric phase in the presence of earth matter effects

In order to calculate non-cyclic GP in the presence of matter, we make use of the formalism given in [35]. In this formalism, the evolution operator which evolves an initial flavor state, in mass eigenstate basis and flavor state basis in a constant matter density are given by

$$U_m(L) = \phi \sum_{a=1}^3 \frac{e^{-iL\lambda_a}}{3\lambda_a^2 + c_1} [(\lambda_a^2 + c_1)I + \lambda_a T + T^2] \quad (14)$$

$$U_f(L) = \phi \sum_{a=1}^3 \frac{e^{-iL\lambda_a}}{3\lambda_a^2 + c_1} [(\lambda_a^2 + c_1)I + \lambda_a \tilde{T} + \tilde{T}^2] \quad (15)$$

respectively. The T matrix has the form

$$\mathbf{T} = \begin{pmatrix} AU_{e1}^2 - \frac{1}{3}A + \frac{1}{3}(E_{12} + E_{13}) & AU_{e1}U_{e2} & AU_{e1}U_{e3} \\ AU_{e1}U_{e2} & AU_{e2}^2 - \frac{1}{3}A + \frac{1}{3}(E_{21} + E_{23}) & AU_{e2}U_{e3} \\ AU_{e1}U_{e3} & AU_{e2}U_{e3} & AU_{e3}^2 - \frac{1}{3}A + \frac{1}{3}(E_{31} + E_{32}) \end{pmatrix}, \quad (16)$$

with $\phi = e^{-\frac{i}{3}L \text{Tr } \mathcal{H}_m}$, $T = \mathcal{H}_m - \text{Tr}[\mathcal{H}_m]/3$, $c_1 = \det \text{Tr}(T^{-1})$ and λ_a are eigenvalues of T , matter potential $A = \pm \sqrt{2} G_F n_e$ (+ for neutrinos, - for anti-neutrinos),

G_F is Fermi constant, n_e is the electron number density, $E_{ij} = E_i - E_j$ and $\tilde{T} = UTU^{-1}$.

If neutrinos travel through a series of constant mat-

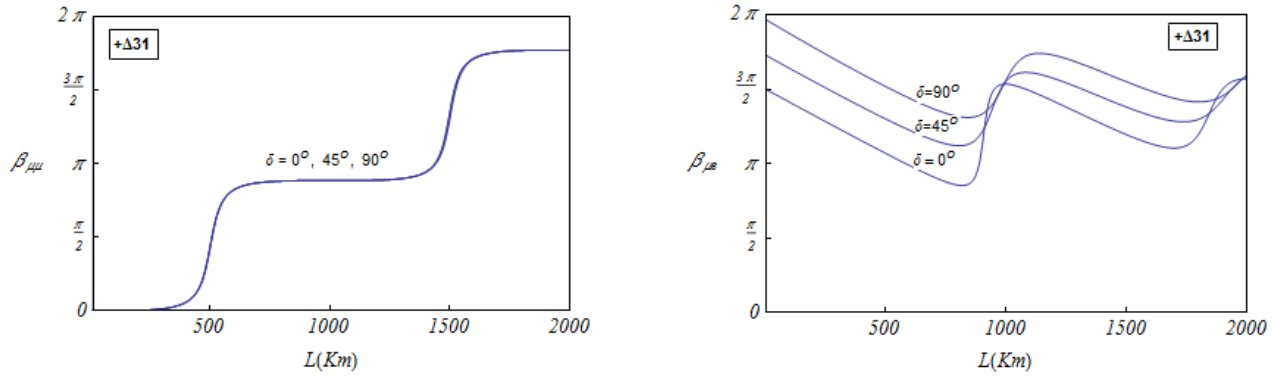


FIG. 1: The left and right panels show the variation of $\beta_{\mu\mu}$ and $\beta_{\mu e}$ with L (in Km), respectively in vacuum for different values of δ . Here neutrino energy $E = 1$ GeV.

ter densities, say $A_1, A_2, A_3, \dots, A_n$, and evolution operators corresponding to these matter densities are $U_1, U_2, U_3, \dots, U_n$ then $U' = U_1 U_2 U_3 \dots U_n$. For Earth's mantle-core-mantle step function density profile, with U_1 and U_2 corresponding to mantle and core of the Earth respectively, U' will become $U' = U_1 U_2 U_1$.

III. Results and discussions

We now study GP for neutrinos produced at various man made facilities such as the reactor and accelerator neutrino experiments. For reactor neutrinos, we consider Daya-Bay and RENO experimental set-ups whereas for accelerator neutrinos, we present results for NO ν A and T2K. Daya-Bay is a China based reactor neutrino experiment[36, 37] which uses a cluster of six nuclear reactors to get $\bar{\nu}_e$ anti-neutrinos with energy range of MeV order and three detectors are placed at distance of units of Km. One far detector is placed at approximately 2 Km distant from the source. This baseline has to face the constant rock matter density with matter potential $A = -1.01 \times 10^{-13}$ eV (negative sign due to $\bar{\nu}_e$). RENO is also a reactor $\bar{\nu}_e$ -experiment. Its baseline is 1.4 Km with energy range in units of MeV. Since initial flavor state in these experiments are $\bar{\nu}_e$, therefore we analyze $\beta_{e\mu}$ component of the GP.

Accelerator neutrino-experiment NO ν A (NuMI Off-Axis ν_e Appearance) experiment uses NuMI-beam of accelerator ν_μ -neutrinos based at Fermilab [38, 39]. The detector is 810 Km (baseline) distant from the source of ν_μ neutrinos in the energy range of 1-10 GeV. Its aim is to measure the small mixing angle θ_{13} , the neutrino-mass hierarchy and the CP -violating phase. This baseline passes through the Earth's crust which has a constant matter density 1.7×10^{-13} eV. T2K is an off-axis experiment [6, 7] which uses a ν_μ - neutrino beam from

Tokai to Kamioka with energy-range of approximately 100 MeV to 1 GeV. It has 295 Km long baseline and passes through a matter density of 1.01×10^{-13} eV. For accelerator neutrinos, we consider $\beta_{\mu e}$ component of the GP.

Fig. 2 shows the variation of $\beta_{e\mu}$ (for Daya-Bay and RENO) and $\beta_{\mu e}$ (for T2K and NO ν A) with neutrino energy, E_n , for different values of CP violating phase δ for an initial anti electron neutrino and muon neutrino beams, respectively. It can be seen from the figure that T2K, Daya-Bay and RENO have neutrino energies and baselines such that the GP can complete at least one cycle. This is because of the factor $(2q-1)\phi = (\frac{\Delta_{32} + \Delta_{31}}{4E\hbar c}) = (\frac{2\Delta_{32} + \Delta_{21}}{4E\hbar c})$, in Eq.(10-13), which plays a leading role in the oscillatory nature of the GP. In accordance with this factor, the value of L/E , corresponding to one cycle, should be ~ 0.99 Km/MeV for $+\Delta_{31}$ and ~ 1.03 Km/MeV for $-\Delta_{31}$. While, the value of L/E is 2950 Km/GeV (2.95 Km/MeV) for T2K (at $L = 295$ Km, $E = 100$ MeV), 2 Km/MeV for Daya-Bay (at $L = 2$ Km, $E = 1$ MeV) and 1.4 Km/MeV for RENO (at $L = 1.4$ Km, $E = 1$ MeV) which is greater than the required L/E value for one cycle, the value of L/E for NO ν A (at $L = 810$ Km, $E = 1$ GeV) ~ 810 Km/GeV (0.81 Km/MeV) is not enough to have at least one cycle. These L/E values for specific experimental set-ups are calculated at their corresponding lowest energy-values of neutrinos and the baseline length.

Further, we find that for L/E corresponding to a cycle, all GP curves corresponding to different values of CP -phase, converge to a single point. We call this point as *cluster point*. This is so because in the limit $\phi \frac{L}{C} \rightarrow \frac{2\pi}{(2q-1)}$, i.e., where one cycle of oscillating feature of GP gets completed, $\beta_{e\mu}$ remains approximately constant for every CP violating phase δ . An interesting thing to observe here is that there are two distinct cluster points corresponding to $+$ and $-$ signs of Δ_{31} . This

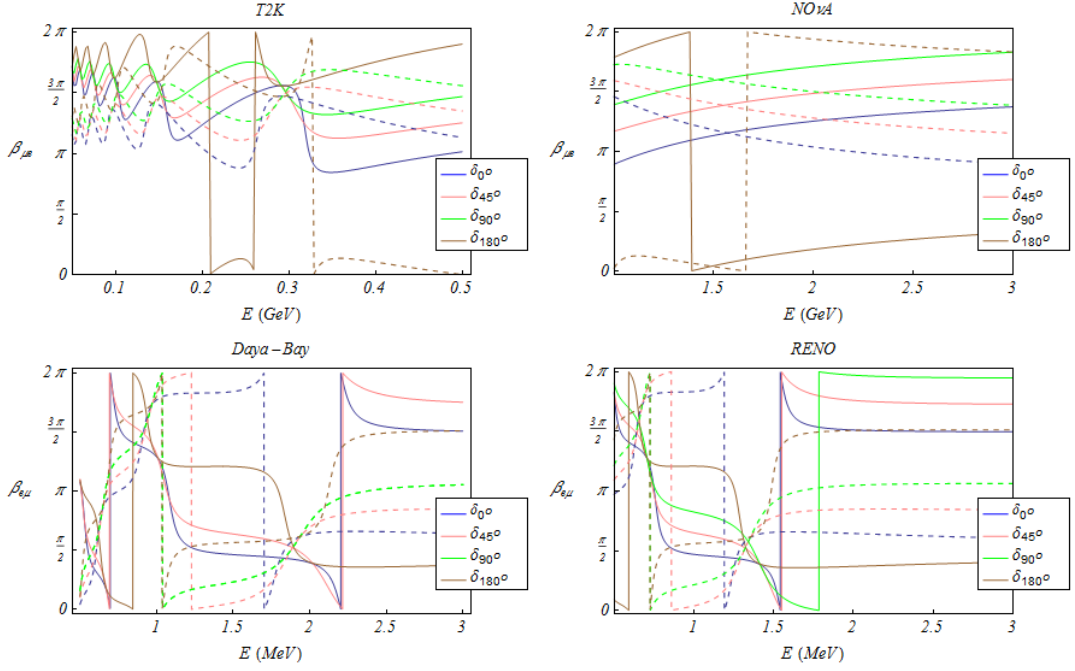


FIG. 2: The left and right panels of top figure depicts $\beta_{\mu e}$ vs E (GeV) plots for T2K and NO ν A experimental set-ups, respectively whereas the left and right panels of the bottom figure shows the variation of $\beta_{e\mu}$ with E (MeV) for Daya-Bay and RENO set-ups, respectively for different CP -violating phases $\delta = 0$ (blue), $\delta = \frac{\pi}{4}$ (pink), $\delta = \frac{\pi}{2}$ (green) and $\delta = \pi$ (brown). Solid and dashed lines in these plots correspond to positive and negative signs of Δ_{31} , respectively.

would thus help in removing the sign ambiguity in Δ_{31} . Therefore if we choose proper energy range, for the fixed baseline length, such that L/E corresponds to a cycle of the GP, then one can disentangle effects of $+$ and $-$ signs of Δ_{31} . For example, for the case of Daya-Bay experiment ($L = 2\text{Km}$), we are getting these cluster points at $E = 1\text{ MeV}$ and 2 MeV . So one has to tune the energy value of anti-neutrinos at these values and by measuring the GP-value, the effect of $+$ and $-$ signs can be disambiguate.

As the baseline length and energies for experiments such as Daya-Bay, RENO and T2K allows atleast one complete cycle of GP, measurement of GP of neutrinos with energy within specific energy range (such that condition for cluster point is satisfied) can enable resolving the sign ambiguity in Δ_{31} . This can be seen from Fig. 3 where predictions for GP for both signs of Δ_{31} are shown for various experimental set-ups. Here L for these set-ups are fixed at their baseline length and E is selected so that L/E corresponds to one cycle of GP. It is obvious from the figure that the predictions for GP are different for $+$ and $-$ signs of Δ_{31} and hence one can determine the sign of Δ_{31} . For Daya-Bay, the predictions for both signs of Δ_{31} are different for all values of CP violating phase for E_n between $(0.95 - 1.02)\text{ MeV}$ and $(1.97 - 2.04)\text{ MeV}$ with L fixed at the baseline length. For RENO

(T2K), the predictions for both signs of Δ_{31} are different for neutrino energy range $(1.38 - 1.42)\text{ MeV}$ ($(95 - 101)\text{ MeV}$). In Fig. 3 dashed and dotted curves represent the case of the lowest and the highest limits of these energy ranges, respectively. Solid curves correspond to that specific energy value at which a cycle of GP gets completed (eg. for RENO, solid curve correspond to $E = 1.4\text{ MeV}$). Hence measurement of GP would be helpful in resolving the degeneracy in Δ_{31} . The points in Fig. 3 where all three curves representing different values of energy, for specific sign of Δ_{31} , intersect because at the value of CP -phase, corresponding to intersection point (say $\delta \sim 195^\circ$ for RENO), the value of GP remains constant in the energy range mentioned above for different experimental set-ups.

Experimental detection of GP usually makes use of interference set-ups. However, in the case of neutrinos, this would not be feasible [40, 41]. Nevertheless, one could envisage an interference experiment in energy space. This idea was used in [19] to show, in the context of two-flavor neutrino oscillations, that the topological (Pancharatnam) phase of the interference term was implicit in the transition probabilities associated with the neutrino oscillations. It was seen to be a consequence of the orthogonality of the two-flavor PMNS matrix. The analysis was made by exploiting the analogy between the

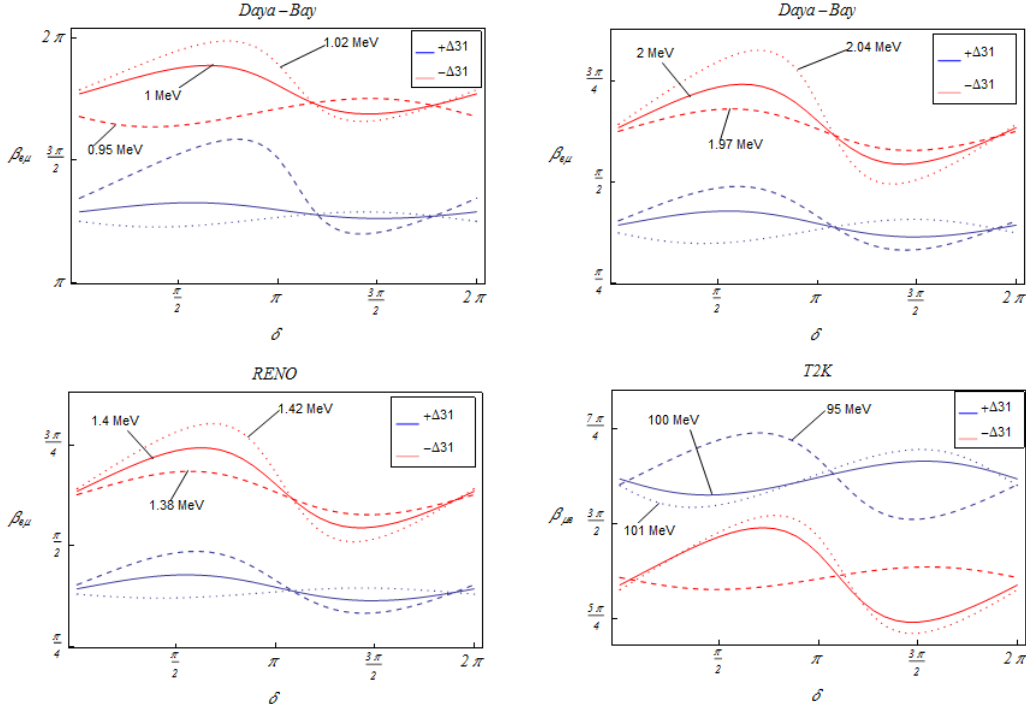


FIG. 3: The left and right panels of top figure show $\beta_{e\mu}$ vs δ plots for Daya-Bay experimental set-up. The left and right panel of bottom figure shows variation of $\beta_{e\mu}$ and $\beta_{\mu e}$ with δ , for RENO and T2K experimental set-ups, respectively. Solid curves in these plots correspond to that specific energy value of neutrino energy at which a cycle of GP gets completed. The solid curves in left and right panel of top figures, for Daya-Bay, correspond to 1 and 2 MeV, respectively. The solid curve for RENO and T2K correspond to 1.4 and 100 MeV, respectively. Dashed and dotted curves correspond to lower and upper limits of neutrino energies for which the GP predictions for both signs of Δ_{31} are different. Blue and red curves correspond to predictions for positive and negative signs of Δ_{31} , respectively.

geometry of two-state neutrino system and polarization states in optics. The discussion of the topological phase was thus made on the Poincaré sphere. For the three-flavor case, discussed here, the space of states is \mathcal{CP}^2 , where \mathcal{CP} stands for complex projective. This reduces to the Poincaré sphere \mathcal{CP}^1 for the two-flavor scenario. The topological phase is due to the curvature of the projective Hilbert space. Here, we have seen that the geometric phase, for the case of flavor transition, is sensitive to the different values of the CP violating parameter. Hence, in analogy with [19], it would be an interesting exercise to study the interference, in energy space, for the three-flavor case and interpret the geometric phase in terms of transition probabilities. This would in turn bring the work discussed here within the realms of possible experimental verification.

IV. Conclusions

We study non-cyclic GP in the context of three flavor neutrino oscillations in the presence of matter and CP vi-

olating effects at various man-made facilities, such as the reactor and accelerator neutrino experimental set-ups. The geometric phase is seen to be sensitive to the CP violating phase in the leptonic sector and the sign ambiguity in Δ_{31} . We analyze GP in the context of two reactor neutrino experimental set-ups, Daya-Bay & RENO and two accelerator experiments, T2K & NO ν A. We find that for experimental facilities where the geometric phase can complete atleast one cycle, all geometric phase curves corresponding to different values of CP violating phase, converge to a single point, called the *cluster point*. There cluster points are distinct for positive and negative signs of Δ_{31} . Thus measurement of geometric phase in experimental set-ups, such as T2K, Daya-Bay and RENO where atleast one complete cycle of GP is possible, would help in resolving the neutrino mass hierarchy problem.

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